

Year 9

Summer Term

Knowledge

Organiser



YEAR 9 — REASONING WITH GEOMETRY...

Enlargement & Similarity

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes.

Keywords

Similar Shapes: shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.

Scale Factor: the multiple describing how much a shape has been enlarged

Enlarge: to change the size of a shape (enlargement is not always making a shape bigger)

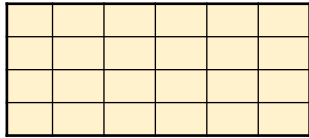
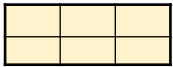
Corresponding: objects (or sides) that appear in the same place in two similar situations.

Image: the picture or visual representation of the shape

Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

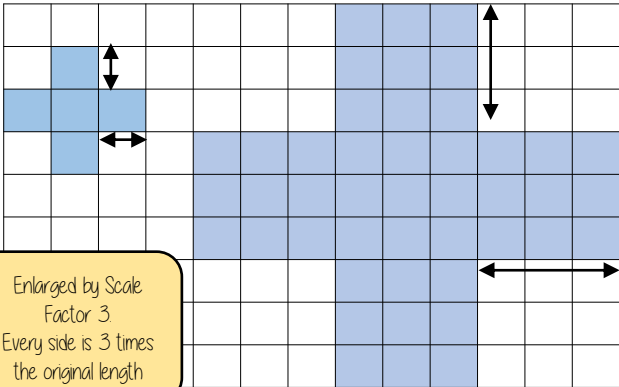
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

Enlarge by a positive scale factor

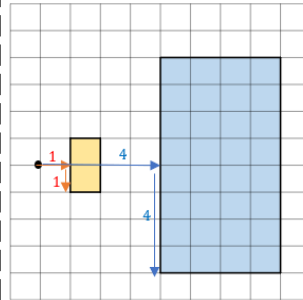
With a scale factor larger than 1 it makes the shape bigger



Enlarged by Scale Factor 3
Every side is 3 times the original length

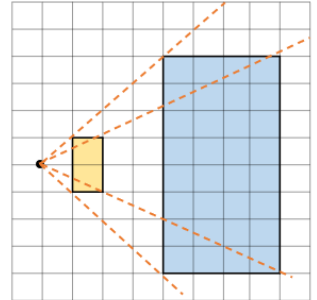
Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

Rays method



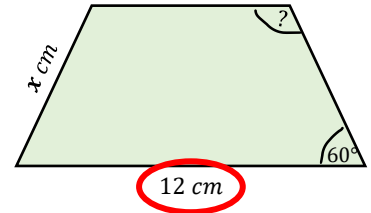
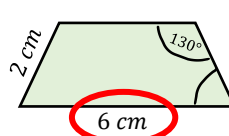
Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

Enlarge by a positive scale factor

Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



Corresponding sides identify the scale factor

$$\frac{12}{6} = 2$$

Scale Factor = 2

Calculate the missing side

Length (corresponding side) \times scale factor

$$2\text{ cm} \times 2$$

$$x = 4\text{ cm}$$

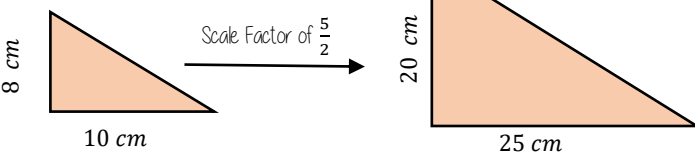
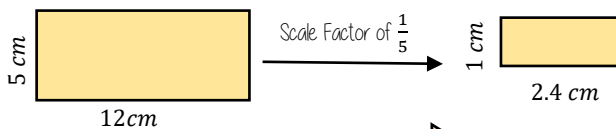
Enlargement does not change angle size

Calculate the missing angle

Corresponding angles remain the same
 130°

Positive fractional scale factor

With a scale factor between 0 and 1 it makes the shape smaller



YEAR 9 — REASONING WITH GEOMETRY...

Solving ratio & proportion problems

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers

Ratio: a ratio shows the relative size of two variables

Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

Inverse proportion: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

R



4 cans of pop = £2.40

4 cans of pop = £2.40
2 cans of pop = £1.20

This multiplier is the same in the same way that this would be for ratio

This is a multiplicative change

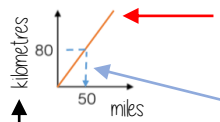
4 cans of pop = £2.40
12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables

R



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare — then find the associated point by using your graph
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Labelling of both axes is vital

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

T	1	2	8
G	40	20	5

Operations: 1 to 2 (x2), 2 to 8 (x4), 40 to 20 (÷2), 20 to 5 (÷4)

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts



Shop A

4 cans for £1.20

£1.20 ÷ 4

Cost per item

1 can is £0.30
Or 30p

Shop B

3 cans for 93p

£0.93 ÷ 3

1 can is £0.31
Or 31p

Shop A is the best value as it is 1p cheaper per can of pop



Shop A

4 cans for £1.20

4 ÷ £1.20

Cost per pound

£1 buys 3.333 cans of pop

3 cans for 93p

3 ÷ £0.93

£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

Sharing a whole into a given ratio

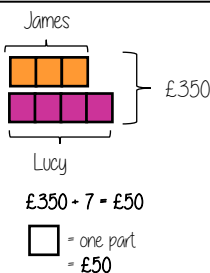
R

James and Lucy share £350 in the ratio 3:4. Work out how much each person earns

Model the Question

James: Lucy

3 : 4



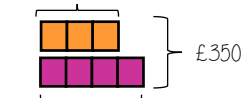
Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

Put back into the question

James: Lucy

James = 3 x £50 = £150



(x50) 3 : 4 (x50)
£150 : £200

Finding a value given 1:n (or n:1)

R

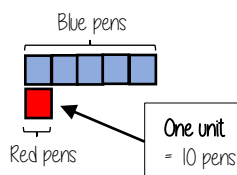
Inside a box are blue and red pens in the ratio 5:1. If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

□ = one part = 10 pens

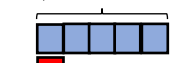


Put back into the question

Blue: Red

(x10) 5 : 1 (x10)
50 : 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

YEAR 9 — REASONING WITH GEOMETRY... Rates

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve speed, distance, time questions
- Use distance time graphs
- Solve density, mass, volume problems
- Solve flow problems
- Use flow graphs
- Interpret rates of change and their units

Keywords

Convert: change

Mass: a measure of how much matter is in an object. Commonly measured by weight

Origin: the coordinate (0, 0)

Volume: the amount of 3D space a shape takes up

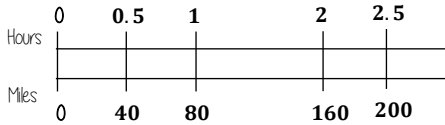
Substitute: putting numbers where letters are — replacing numbers into a formula

Speed, Distance, Time

'per' for every
e.g. 80 miles per hour (mph)
Travel 80 miles every hour

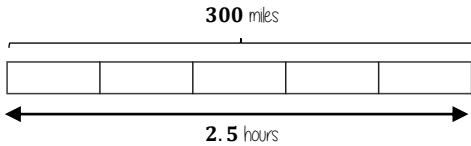
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



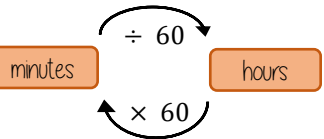
e.g. A boat travels at a constant speed for 2.5 hours
It travels 300 miles.

Bar models can help to calculate mph



Each part is half an hour
Each part is 60 miles

Speed, Distance, Time



Before calculations — make sure you are working in the same units as the speed

Learn or learn how to rearrange the formula for speed, distance and time

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

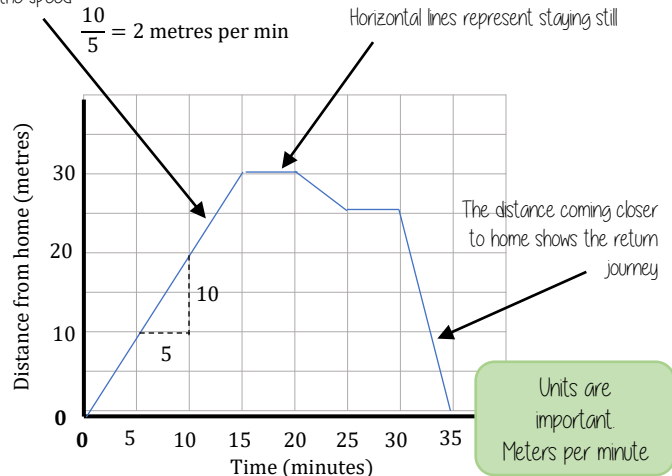
$$\text{distance} = \text{speed} \times \text{time}$$

Substitute in the variables given

Distance — Time graphs

The steeper a gradient the faster the speed

Gradient = speed

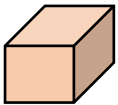


Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$



$$\text{volume of prism} = \text{Area of cross section} \times \text{Depth}$$



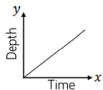
Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



Units are important
Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

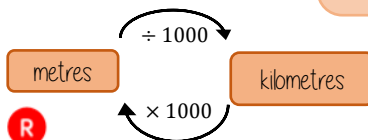
Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles per hour

Exchange rates: euros per pounds

Density: mass per volume



YEAR 9 — REPRESENTATIONS...

Probability

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

Keywords

Probability: the chance that something will happen

Relative Frequency: how often something happens divided by the outcomes

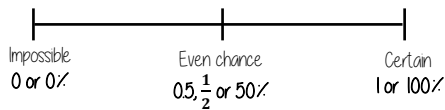
Independent: an event that is not effected by any other events.

Chance: the likelihood of a particular outcome.

Event: the outcome of a probability — a set of possible outcomes.

Biased: a built in error that makes all values wrong by a certain amount.

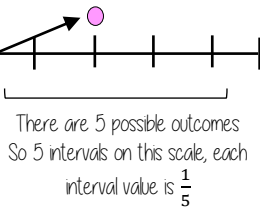
The probability scale



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability



R

Single event probability

Probability is always a value between 0 and 1



The probability of getting a blue ball is $\frac{1}{5}$
 \therefore The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$$



R

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections

$$\text{Relative frequency} \times \text{Number of times} \\ 0.3 \times 100 = 30$$

Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

An experiment is carried out 400 times

Show that dark chocolate is expected to be selected 60 times

$$0.15 \times 400 = 60$$

Independent events



The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

$$\text{Probability of event 1} \times \text{Probability of event 2}$$



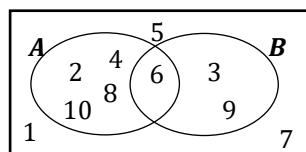
$$P(5) = \frac{1}{6} \quad P(R) = \frac{1}{4}$$

Find the probability of getting a 5 and a red

$$P(5 \text{ and } R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$$

Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables



	Car	Bus	Wak	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from tossing a coin

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

YEAR 9 — REPRESENTATIONS... Algebraic Representation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw quadratic graphs
- Interpret quadratic graphs
- Interpret other graphs including reciprocals
- Represent inequalities

Keywords

Quadratic: a curved graph with the highest power being 2. Square power.

Inequality: makes a non equal comparison between two numbers

Reciprocal: a reciprocal is 1 divided by the number

Cubic: a curved graph with the highest power being 3. Cubic power.

Origin: the coordinate (0, 0)

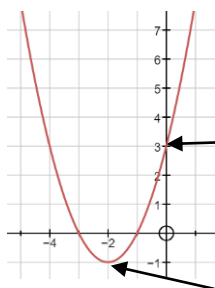
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$$y = x^2 + 4x + 3$$

If x^2 is the highest power in your equation then you have a quadratic graph

It will have a parabola shape



Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting (-3, 0)

Plot all of the coordinate pairs and join the points with a curve (freehand)

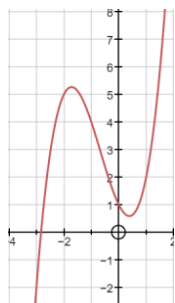
Quadratic graphs are always symmetrical with the turning point in the middle

Interpret other graphs

Cubic Graphs

$$y = x^3 + 2x^2 - 2x + 1$$

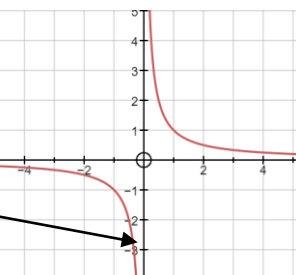
If x^3 is the highest power in your equation then you have a cubic graph



Reciprocal Graphs

$$y = \frac{1}{x}$$

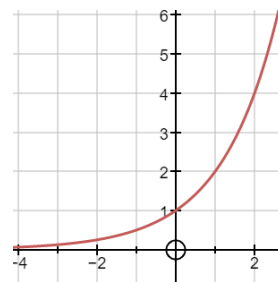
Reciprocal graphs never touch the y axis
This is because x cannot be 0
This is an asymptote



Exponential Graphs

$$y = 2^x$$

Exponential graphs have a power of x

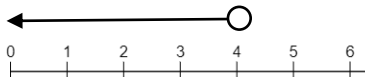


Represent Inequalities

Multiple methods of representing inequalities

$$x < 4$$

All values are less than 4



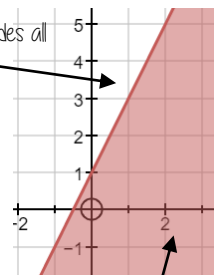
The shaded area indicates all possible values of x



The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$$y \geq 2x + 1$$



The shaded area indicates all possible solutions to this inequality